Math 141 **Test #2 Solutions** **Fall 2004**

1. Use the limit process:

$$\begin{aligned}
\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} &= \lim_{\Delta x\to 0}\frac{(2(x+\Delta x)^2+3(x+\Delta x))-(2x^2+3x)}{\Delta x}\\
&= \lim_{\Delta x\to 0}\frac{2x^2+4x\Delta x+2(\Delta x)^2+3x+3\Delta x-2x^2-3x)}{\Delta x}\\
&= \lim_{\Delta x\to 0}\frac{4x\Delta x+2(\Delta x)^2+3\Delta x}{\Delta x}\\
&= \lim_{\Delta x\to 0}(4x+2\Delta x+3)\\
&= 4x+3.
\end{aligned}$$

2. Compute $f'(x)=3x^2$. Then evaluate: $f'(2)=3(2^2)=12$. This is the slope of the line. Therefore, the equation is

$$y-8=12(x-2).$$

3. Use the rules of differentiation.

   (a) $f'(x)=20x^3+16x-10$.

   (b) $f'(x)=\frac{1}{2\sqrt{x}}$.

   (c) Write $f(x)=5x^{-2}$. Then $f'(x)=-10x^{-3}=-\frac{10}{x^3}$.

   (d) $f'(x)=2x\sin x+x^2\cos x$.

   (e) $f'(x)=\frac{(3x+2)(2x)-(x^2+1)(3)}{(3x+2)^2}$

   (f) $f'(x)=-2\csc 2x$ or write $f(x)=\frac{\cos 2x}{\sin 2x}$ and use the quotient rule:

$$f'(x)=\frac{\sin 2x(-2\sin 2x)-\cos 2x(2\cos 2x)}{\sin^2 2x}=-\frac{2}{\sin^2 2x}=-2\csc 2x.$$

   (g) $f'(x)=10(3x^5+1)^9(15x^4)$

4. Let $y=8x^4+6x^3-2x$. Then

$$dy/dx=32x^3+18x^2-2$$

and

$$d^2y/dx^2=96x^2+36x.$$

5. Differentiate $x + y = xy$ implicitly to get $1 + y' = y + xy'$. Then solve for y':

$$y' = \frac{y-1}{1-x}.$$

To find y'', you may differentiate this equation or you may differentiate implicitly the earlier equation $1 + y' = y + xy'$:

$$y'' = y' + y' + xy''.$$

Solve for y'': $y'' = \frac{2y'}{1-x}$. Then substitute into this the previous expression for y' to get an answer in terms of only x and y:

$$y'' = \frac{2(y-1)}{(1-x)^2}.$$

6. First, since $r = h$, we may rewrite the volume formula as

$$V = \frac{1}{3}\pi h^3.$$

Then differentiate with respect to t:

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}.$$

Now substitute $h = 3$ and $\frac{dV}{dt} = 5$ and solve for $\frac{dh}{dt}$.

7. Let x be the horizontal distance and h the vertical distance to the ends of the ladder. Then

$$x^2 + h^2 = 15^2.$$

Differentiate:

$$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 0.$$

When $x = 9$, we find that $h = 12$. So substitute $x = 9$, $h = 12$, and $\frac{dx}{dt} = 3$ and solve for $\frac{dh}{dt}$.